



CONTROLO INTEGRADO DA PRODUÇÃO

MODELAÇÃO E CONTROLO DE SISTEMAS DE PRODUÇÃO

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Programa

- 1. Introdução: modelos de manufatura e de serviços**
Modelos de Sistemas de Manufatura. Modelos de Sistemas na Área dos Serviços.
- 2. Planeamento e escalonamento em sistemas de manufatura**
Planeamento de projectos. Escalonamento de máquinas e escalonamento *job shop*. Escalonamento de sistemas de montagem flexíveis. Escalonamento por lotes. Planeamento e escalonamento em cadeias de abastecimento.
- 3. Planeamento e escalonamento na área dos serviços**
Escalonamento por intervalos, reservas e escalonamento temporal. Escalonamento temporal em desportos e lazer. Planeamento, escalonamento e horários em transportes. Escalonamento de equipas de trabalho.
- 4. Desenvolvimento e implementação de sistemas de produção**
Desenvolvimento e implementação de sistemas. Conceitos avançados em concepção de sistemas. Futuro da área do controlo integrado da produção.

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Programa detalhado

1 – Introdução: modelos de manufatura e serviços

□ Introdução

- Definição de controlo integrado da produção. Papel e impacto do planeamento e escalonamento em sistemas de produção. Funções do controlo integrado da produção e do planeamento e escalonamento numa empresa.

□ Modelos de Sistemas de Manufatura

- Introdução. Trabalhos, máquinas e recursos. Processamento das características e das restrições em sistemas de manufatura. Objectivos e medidas de desempenho.

□ Modelos de Sistemas na Área dos Serviços

- Introdução. Actividades e recursos em serviços. Características operacionais e restrições. Objectivos e medidas de desempenho.

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Programa detalhado (2)

2 – Planeamento e escalonamento em sistemas de manufatura

□ Planeamento de projectos

- Método do Caminho Crítico (CPM). Avaliação do programa e técnica de revisão (PERT). Tempo vs. custo: métodos lineares e não-lineares. Escalonamento de projecto com restrições de equipas de trabalho. Exemplo: sistema de escalonamento de um projecto para a indústria nuclear.

□ Escalonamento de máquinas e escalonamento *job shop*

- Máquina única e modelo de máquinas paralelas. Programação matemática aplicada a *job shop*. Heurística *shifting bottleneck* para *job shop*. Programação com restrições aplicada a *job shop*. LEKIN: Exemplo de um sistema genérico para escalonamento *job shop*.

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Programa detalhado (3)

□ Escalonamento de sistemas de montagem flexíveis

- Sequenciação de sistemas de montagem em linhas paralelas. Sequenciação de sistemas de montagem com estações de trabalho. Escalonamento de sistemas de fluxo flexível (*flexible flow shop*) com saltos. Exemplo: modelo misto de sequenciação de montagem na Toyota.

□ Escalonamento por lote

- Escalonamento com um tipo de produto. Escalonamentos rotativos com vários tipos de produtos. Escalonamentos arbitrários com vários tipos de produtos. Modelos genéricos de escalonamento por lotes. Exemplo: planeamento e escalonamento de vários produtos na Owens-Corning Fiberglas.

□ Planeamento e escalonamento em cadeias de abastecimento

- Definição de cadeia de abastecimento. Configurações possíveis. Métodos de planeamento e escalonamento em cadeias de abastecimento. Modelos de planeamento a médio prazo e a curto prazo para cadeias de abastecimento. Exemplo de implementação na Carlsberg Denmark.

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Programa detalhado (4)

3 – Planeamento e escalonamento na área dos serviços

□ Escalonamento por intervalos, reservas e escalonamento temporal

- Sistemas de reservas sem folga e com folga. Escalonamento temporal de equipas de trabalho com restrições. Escalonamento temporal com restrições de operadores ou de ferramentas. Exemplo: atribuição de disciplinas a salas na Universidade de Berkeley.

□ Escalonamento temporal em desportos e lazer

- Escalonamento temporal em competições desportivas. Escalonamento de competições com programação com restrições e por procura local. Escalonamento de programas televisivos por cabo. Exemplo: escalonamento de uma competição de basquetebol.

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Programa detalhado (5)

- ❑ **Planeamento, escalonamento e horários em transportes**
 - Escalonamento de frotas. Escalonamento e planeamento de rotas de aeronaves. Horários de comboios. Exemplo: concepção e implementação de sistemas de transporte: Carmen Systems.
- ❑ **Escalonamento de equipas de trabalho**
 - Introdução. Escalonamento de dias de folga de trabalhadores. Escalonamento de turnos. Problema de turnos de trabalho cíclicos. Escalonamento de tripulações e equipas de trabalho. Exemplo: escalonamento de operadores num *call center*.

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Programa detalhado (6)

- 4 – Desenvolvimento e implementação de sistemas de produção**
- ❑ **Desenvolvimento e implementação de sistemas**
 - Arquitectura de sistemas. Bases de dados, bases de objectos e bases de conhecimento. Módulos para gerar planeamentos e escalonamentos. Interfaces com o utilizador e optimização interactiva. Sistemas genéricos e sistemas específicos para uma dada aplicação. Aspectos de implementação e manutenção.
- ❑ **Conceitos avançados em concepção de sistemas**
 - Robustez e tomada de decisão reactiva. Mecanismos de aprendizagem. Motores de planeamento e escalonamento, e bibliotecas de algoritmos. Sistemas reconfiguráveis. Planeamento baseado na *web* e sistemas de escalonamento.
- ❑ **Futuro da área do controlo integrado da produção**
 - Futuro do planeamento e escalonamento em sistemas de manufactura. Futuro do planeamento e escalonamento em serviços. Métodos de optimização. Desenvolvimento de sistemas.

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Bibliografia

- ❑ Pinedo, M., *Scheduling Operations, Manufacturing and Services*, Springer, 2005.
- ❑ Pinedo, M., *Scheduling: Theory, Algorithms, and Systems*, Prentice-Hall Inc., 2002.
- ❑ N. Viswanadham, Y. Narahari, *Performance modeling of automated manufacturing systems*, Prentice Hall, 1992.
- ❑ M. P. Groover, Automation, *Production Systems and Computer Integrated Manufacturing*, Prentice Hall, 2001.
- ❑ R.D. Klafter, T.A. Chmielewski & M. Negin. *Robotic Engineering - An Integrated Approach*. Prentice Hall International, Inc., 1989.

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Avaliação de conhecimentos

- ❑ Exame (50%) e Projecto (50%)
- ❑ Possibilidade de fazer mini-testes ao longo do semestre, dispensando-se de exame.
- **Projecto:** grupos de 2 alunos no máximo (apresentado oralmente).
 - Na célula flexível de produção:
 - <http://193.136.103.195/view/index.shtml>
 - e/ou em colaboração com indústria / serviços:
 - Planeamento optimizado de produção
 - Simulação de layout de fábrica real

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INTRODUCTION



Planning and Scheduling

- ❑ Decision-making processes used in many manufacturing and service industries.
- ❑ Applied in procurement and production, transportation and distribution, information processing and communication.
- ❑ Planning and scheduling rely on **mathematical techniques** and **heuristic methods** to optimize allocation of limited resources to activities.

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Example: system installation project

- ❖ **Example:** Procurement, installation and testing of a large computer system.
- ❑ **Tasks:** evaluation and selection of hardware, software development, recruitment and training of personnel, system testing and debugging, etc.
- ❑ **Goal:** complete the project in minimum time, considering the precedence between tasks.

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Example: job shop manufacturing

- ❖ **Example:** semiconductor manufacturing facility.
- ❑ **Tasks:** wafer fabrication, wafer probe, assembly and final testing (highly specialized manufacturing).
- ❑ **Goals:** meet as many due dates as possible, while maximizing throughput.
- ❑ Wafer fabrication consists of several layers, requiring the repetition of operations several times.
- ❑ Number of orders in the system are usually hundreds and each has its own release date and due date.

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Example: flexible assembly system

- ❖ **Example:** automobile assembly line.
- ❑ **Tasks:** producing different models, belonging to a small family of cars.
- ❑ **Goals:** maximizing throughput balancing the workload at each station.
- ❑ Family of cars can include two-door coupe, four-door sedan and stationwagon.
- ❑ A bottleneck is the paint shop: color changing is a time consuming process.

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Example: production planning

- ❖ **Example:** production planning in a paper mill.
- ❑ **Tasks:** Each machine produces various types of paper, characterized by basis weight, grade and color.
- ❑ **Goals:** maximize throughput, minimizing inventory costs.
- ❑ **Input:** wood fiber and pulp. **Output:** rolls of paper.
- ❑ Paper machines: 50 to 100 million euros each.
- ❑ Production plans drawn on an *annual* basis. Cycles of production of 2 weeks.

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Example: supply-chain

- ❖ **Example:** planning an scheduling in a supply-chain.
- ❑ **Tasks:** material or goods are moved from one facility to another (in a network of facilities).
- ❑ **Goals:** minimize the total costs (production, transportation and inventory holding costs).
- ❑ Paper mill is included in a network of production facilities: timberland, paper mills, converting facilities, distribution centers and retailers or end-consumers.
- ❑ More value is added to the product in each stage of the supply chain.

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Example: reservation system

- ❑ Scheduling problems of a manufacturer are similar to scheduling problems at services (e.g. car rental, hotels).
- ❖ **Example:** car rental agency.
- ❑ **Tasks:** decide to provide or not cars to clients. Reservations for very short periods can be denied.
- ❑ **Goals:** maximize number of days cars are rented out.
- ❑ Agency maintains a fleet of various types of cars.

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Example: scheduling in sports

- ❖ **Example:** tournament of a soccer (football) league.
- ❑ **Tasks:** to schedule the games over a fixed set of rounds.
- ❑ **Goals:** to create an ideal schedule that alternates between games at home and games away.
- ❑ Some possible constraints:
 - If a city has two teams, in each round one team should play at home and the other team should play away.
 - If two teams are very strong, the other teams should not face these two teams in consecutive rounds.

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Example: planning in transportation

- ❖ **Example:** routing and scheduling of airplanes.
- ❑ **Tasks:** estimate profits of assigning a type of aircraft to a flight leg.
- ❑ **Goals:** combine the different flight legs into the round trips that can be assigned to airplanes.
- ❑ **Flight:** characterized by **origin**, **destination** and **scheduled departure time**.
- ❑ Information with customer demand for any given flight is available.

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Example: scheduling of personnel

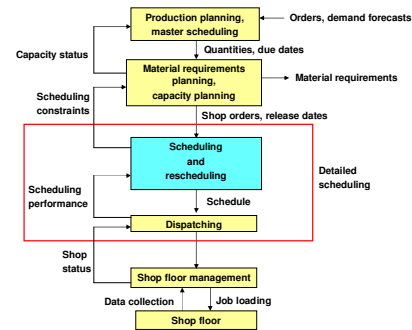
- ❖ **Example:** scheduling of nurses in a hospital.
- ❑ **Goal:** develop shift assignments so that all daily requirement are met and the constraints satisfied at minimal cost.
- ❑ Number of nurses required on week days is usually more than on weekends.
- ❑ The same happens with day shifts and night shifts.
- ❑ State and federal regulations and union rules must may provide additional constraints.

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Information flow in manufacturing

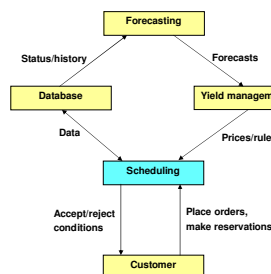


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Information flow in services



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MANUFACTURING MODELS



Manufacturing models

- ❑ In manufacturing models:
 - Resource is called a “**machine**”
 - Task is called as “**job**”
- ❑ A job may be a single operation or a collection of operations to be done in several different machines.
- ❑ There are five classes of manufacturing models, which are described in the following.

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Project planning and scheduling

- ❑ Project scheduling is important for large projects
- ❑ A large project consists of a number of jobs with **precedence constraints**.
- ❖ Example: construction of an aircraft, large consulting project.
- ❑ **Goal**: minimize completion time of last job (**makespan**)
- ❑ The **critical path** (set of jobs that determine the makespan) can be identified.

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Job shop models

- ❑ Job shop scheduling (include single machine and parallel machine models)

Jobs	Machine Sequence
1	1, 2, 3
2	2, 1, 4, 3
3	1, 2, 4

- ❑ Minimize makespan or the number of late jobs
- ❑ Mostly for make-to-order manufacturing systems
- ❑ Also in services

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Flexible Manufacturing Systems

- ❑ Production systems with automated material handling.
- ❑ Material handling or conveyor system controls the movement of jobs and timing of their processing.
- ❑ Mostly for mass production systems.
- ❑ Maximize throughput.
- ❖ **Examples**: automotive industry and consumer electronics industry.

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Lot scheduling

- ❑ For medium and long term production planning.
- ❑ Processes are continuous.
- ❑ Switching between products incurs a setup cost.
- ❑ Minimize total inventory and setup costs.
- ❖ **Examples**: process industries, e.g. oil refineries, paper mills.

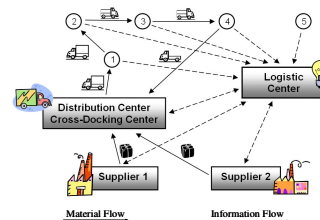
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Supply-chain models

- ❑ In general, are an integration of job-shop and lot scheduling, including transportation costs.



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Manufacturing models revisited

- ❑ **Discrete models:** project scheduling, job shop or flexible assembly systems.
 - Formulated as an **integer programming** or disjunctive programming.
- ❑ **Continuous models:** lot scheduling.
 - Formulated as a **linear or nonlinear programming**

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Jobs and machines

- machines $i = 1, \dots, m$
- jobs $j = 1, \dots, n$

Static data:

- ❑ **Processing time** p_{ij} – processing time of job j on machine i
 - $Q_j = 1/p_j$, machine's production rate of job j
- ❑ **Release date** r_j – earliest time at which job j can start its processing
- ❑ **Due date** d_j – committed shipping or completion date of job j
- ❑ **Weight** w_j – importance of job j relative to the other jobs in the system

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Modeling parameters

Dynamic data:

- ❑ **Starting time** S_{ij} – time when job j starts its processing on machine i
- ❑ **Completion time** C_{ij} – time when job j is completed on machine i

Model representation:

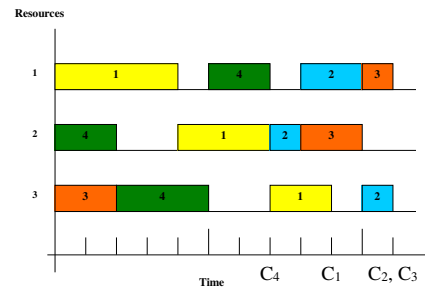
machine configuration | characteristics | objectives
 $\alpha | \beta | \gamma$

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Completion time



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Machine configuration

- ❑ **Single machine:**

- ❑ When there is a single **bottleneck** in a multi-machine environment, that bottleneck is scheduled first.
- ❑ **Earliest Due Date (EDD)** – orders the jobs in increasing order of their due dates.
 - Minimize maximum lateness among all jobs
- ❑ **Short Processing Time first (SPT)** – minimize the average number of jobs waiting for processing.

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Machine configuration

- ❑ **Parallel machine:**

- ❑ Machines do **not** have to be identical.
- ❑ “Machines” can be old or new, or can be people.

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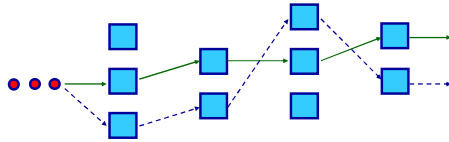
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Machine configuration

❑ **Flow shop:**

❑ **Flexible flow shop:**



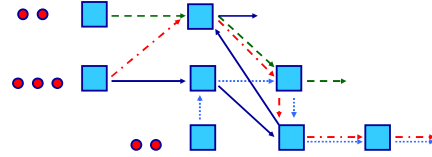
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Machine configuration

❑ **Job shop:** (can have recirculation)



❑ **Flexible job shop** (several machines in parallel)

❑ **Supply-chain**

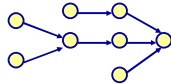
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Processing characteristics

❑ **Precedence constraints** – one or more jobs may have to be completed before another job is allowed to start its processing



❑ **Machine eligibility constraints** – in parallel machine environment, M_j denotes the subset of machines that can process job j

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Processing characteristics

❑ **Workforce constraints**

- workforce consists of several pools
- each pool consists of operators with a specific skill set
- number of operators in pool l is denoted by W_l

❑ **Routing constraints** – specify the operations for each job and the machines at which these operations must be processed.

❑ **Material handling constraints** – are fixed for automated (e.g., robotized) work centers and adjustable for manual tasks.

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Processing characteristics

❑ **Sequence-dependent setup times and costs**

- length of setup (reconfiguration or cleaning) depends on jobs
- s_{ijk} : setup time for processing job k after job j on machine i
- c_{ijk} : setup costs, e.g. waste of raw material, labor

❑ **Storage space and waiting time constraints** – amount of space available for Work-In-Progress (WIP) is limited.



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Processing characteristics

❑ **Make-to-Stock and Make-to-Order**

- D_j – demand rate fixed and constant. Items are produced for inventory and do not have tight dates.
- Make-to-Order jobs have specific due dates; amount produced is determined by the customer.

❑ **Preemptions** – interrupt the processing of a job to process another one with higher priority.

- Preemptive resume – processing done is not lost
- Preemptive repeat – processing must be completely repeated

❑ **Transportation Constraints, etc.**

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Performance Measures and Objectives

- ❑ **Throughput** – is frequently determined by the bottleneck machines, for which the utilization should be maximized.

- ❑ **Makespan**

$$C_{\max} = \max(C_1, C_2, \dots, C_n)$$

- where C_j is the completion time of job j .
- Minimizing makespan tends to maximize throughput and balance load.



Performance Measures and Objectives

- ❑ **Due date related objectives**

- **Lateness**

$$L_j = C_j - d_j$$

- where d_j is the due date of job j .

- **Maximum lateness** (minimize worst performance)

$$L_{\max} = \max(L_1, \dots, L_n)$$



Performance Measures and Objectives

- **Tardiness**

$$T_j = \max(C_j - d_j, 0)$$

- Objective function

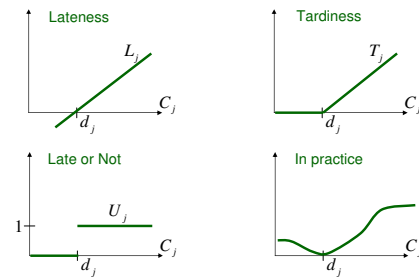
$$\sum_{j=1}^n T_j$$

- **Weighted Tardiness**

$$\sum_{j=1}^n w_j T_j$$



Due date related objectives



Performance Measures and Objectives

- ❑ **Work-In-Process inventory costs**

- Minimizing WIP also minimizes *average throughput (lead) time*, which is the time it takes a job to transverse the system.
- Equivalent to minimize the average number of jobs in the system.
- Minimizing average throughput time is closely related to minimize the sum of completion times:

$$\sum_{j=1}^n C_j \quad \sum_{j=1}^n w_j C_j$$



Others costs and concepts

- ❑ **Setup costs**

- ❑ **Finished goods inventory costs**

- ❑ **Transportation costs**

- ❑ In **Just-In-Time (JIT)** concepts, it is important to minimize the **total earliness**.

- A job should be completed just before its committed shipping, avoiding inventory and handling costs.

- ❑ **Robustness.** A schedule is **robust** when the necessary changes in case of disruption (e.g. machine breakdown, rush order) are minimal.

SERVICE MODELS



Introduction

- ❑ Impossible to “store” goods
 - It is not possible to “get back” the lost time in a hotel room.
- ❑ Resource availability (e.g. people, rooms or trucks) often varies
 - May even be part of the objective function
- ❑ Saying “no” to a customer is common
 - “No available seats on that flight” (even if there are some!)
 - Try to book a restaurant for 8 or 9 PM

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Reservation systems and timetabling

- ❑ **Reservation systems**
 - A job j has a **duration** p_j and the starting and completion times are usually fixed in advance.
 - ❖ **Example:** in a car rental agency, a job is the reservation of a car for a given period.
- ❑ **Timetabling (rostering)**
 - A job or activity j with a duration p_j , which has to be scheduled in a time window in the interval:
[earliest possible starting time r_j , latest possible completion time d_j]
 - **Examples:** exam scheduling, scheduling operating rooms

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Service models

- ❑ **Tournament scheduling and broadcast television models**
 - Tournament scheduling – *parallel machine* problem, where all the jobs have the same processing time.
- ❑ **Transportation scheduling**
 - ❖ **Examples:** airlines, railroads, shipping.
 - **Job** – trip or flight leg; **machine** – ship, plane or vehicle.
 - Trip k incurs a cost c_k and generates a profit π_k .
 - Objective: minimize total cost or maximize total profit.

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Workforce scheduling

- ❑ **Shift scheduling** in service facilities
 - ❖ **Example:** call center
 - Time interval i requires a staffing of b_i (integer).
 - Objective: minimize total cost.
- ❑ **Crew scheduling** in transportation.
 - Depends on the specific tasks to be done
 - Crew scheduling is often intertwined with other schedules (e.g. routing and scheduling of planes or trucks).

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Activities

- ❑ **Examples of activities:**
 - meetings to be attended by certain people
 - game to be played by 2 teams
 - flight leg to be covered by a plane
 - personnel position to be occupied in a given time period
- ❑ **Data:**
 - duration → processing time p_{ij}
 - earliest possible start time → release time r_j
 - latest possible finishing time → due date d_j
 - priority level → weight w_j

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Resources

- ❑ **Machines**: classroom, hotel, rental car, stadium, operating room, plane, ship, airport gate, dock, railroad track, person (nurse/pilot)
- ❑ **Synchronization** of resources may be important
 - Need a plane and a pilot
 - Classroom, video projector equipment, professor, students
- ❑ **Characteristics of resources**
 - **Classroom**: capacity, equipment, cost, accessibility.
 - **Truck**: capacity, refrigeration, speed
 - **Person**: specialist (surgeon, nurse) with skills (languages)

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Operational characteristics

- ❑ **Time windows** (release dates and due dates)
- ❑ **Capacity requirements and constraints**
- ❑ **Preemptions** – many activities are difficult to preempt (e.g. operations flight leg or game).
- ❑ **Setup times and turnaround times**
 - Setups between consecutive meetings or trips.
- ❖ **Example**: Runway in an airport is a resource, takeoff and landing are activities. Necessary idle time between activities.

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Operational characteristics

- ❑ **Operator and tooling requirements**
- ❑ **Workforce scheduling constraints**
 - Shift patterns, break requirements
 - Union and safety rules

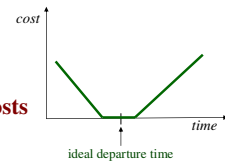
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Objectives

- ❑ Combination of two types of objectives: one concerning the *timings* and other concerning the *utilization of resources*.
- ❑ **General objective**: minimize the total cost of all the assignments.
- ❑ **Makespan**
- ❑ **Setup costs**
- ❑ **Earliness and Tardiness costs**
- ❑ **Personnel costs**



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Computational complexity

- ❑ **Easy problems**:
 - Sort n numbers
 - Solve a system of linear equations
- ❑ **Hard problems**:
 - Schedule a factory, deliver packages, schedule buses, ...

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Computational complexity

- ❑ $f(n)$: the number of “basic operations” needed to solve the problem with input size n
- ❑ **Easy**: $f(n)$ is polynomial in n
 - $\mathcal{O}(n)$, $\mathcal{O}(n \log n)$, $\mathcal{O}(n^2)$, ...
- ❑ **Hard**: $f(n)$ is **exponential** in n
 - $\mathcal{O}(2^n)$, ...

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Hard vs Easy

$\mathcal{O}(n)$	$\mathcal{O}(n \log n)$	$\mathcal{O}(n^2)$	$\mathcal{O}(2^n)$
1	0	1	2
10	10	100	1024
20	26	400	
50	85	2500	
100	200	10000	
1000	3000	1,000,000	

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Hard vs Easy

$\mathcal{O}(n)$	$\mathcal{O}(n \log n)$	$\mathcal{O}(n^2)$	$\mathcal{O}(2^n)$
1	0	1	2
10	10	100	1024
20	26	400	1048576
50	85	2500	1,125,899,906,842,624
100	200	10000	
1000	3000	1,000,000	

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Hard vs Easy

$\mathcal{O}(n)$	$\mathcal{O}(n \log n)$	$\mathcal{O}(n^2)$	$\mathcal{O}(2^n)$
1	0	1	2
10	10	100	1024
20	26	400	1048576
50	85	2500	1,125,899,906,842,624
100	200	10000	1.268×10^{30}
1000	3000	1,000,000	1.072×10^{301}

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Hard vs Easy

- ❑ 10^{301} operations required in worst case
- ❑ Age of universe: 10^{18} seconds
- ❑ Fastest Computer today: 10^{14} op/sec
- ❑ If it is going to take 10^{250} times the age of the universe to schedule a factory?
- *May be it is possible to do it in a reasonable time in most cases!*
- *May be a good (but not the best) solution can be obtained in a reasonable amount of time!*

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P and NP problems

- ❑ The efficiency of an algorithm is measured by the maximum (worst-case) number of computational steps needed to obtain an optimal solution.
- ❑ Problems which have a known **polynomial** algorithm are said to be in **class P**; the effort of the algorithm is bounded by a polynomial function of the size of the problem.
- ❑ For **NP (non-deterministic polynomial problems)** no simple algorithm yields optimal solutions in a limited amount of computer time.

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